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Coordinated activity and common ground during group problem solving in biology[☆]

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ABSTRACT

For groupwork to support learning it requires that students establish mutual knowledge, which is information that becomes shared among all group members. In this study, we analyzed the verbal interactions of two groups of high school students working on a color vision activity. Students within a group each wore different color-filtering goggles, and compared their individual perceptions to identify various colors they viewed together. Because the color-filtering goggles gave each student different information related to the task, sharing knowledge was necessary for successful color recognition. Our analysis was guided by the questions, *how did students establish mutual knowledge through their talk?* And, *what types of knowledge were shared through these processes?* We found that students were more inclined to explain—including providing warrants for their claims—when they used discussion moves such as asking each other questions, reacting to each other's statements, and incorporating multiple perspectives. Practical interventions designed to teach students to work productively in groups should attend to expectations around the content of students' talk in addition to the range of talk moves that students can use to contribute to a discussion.

1. Introduction

Although groupwork is valuable for its potential to facilitate collaboration among students (Cohen, 1994), this outcome is not always achieved. Collaboration is a complex phenomenon characterized by a range of factors, including the participation of different group members (DeJarnette & González, 2015; Esmonde & Langer-Osuna, 2013; Wood, 2013; Zahner & Moschkovich, 2010), the extent to which group members acknowledge and value their peers' contributions (DeJarnette, 2022; Langer-Osuna, 2016), and the ways in which students' social activity supports learning (Barron, 2000, 2003; Francisco, 2013; Webb et al., 2002). To be productive, group work requires more than a group of students seated in close proximity. There is no single path by which group work fosters collaboration, but it is clear that collaboration requires students to establish some degree of coordinated activity.

Mutual knowledge, also described as common ground, is necessary for individuals to engage in collaborative work or any joint

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meaning making (Schwartz, 1995). Mutual knowledge in the context of group work refers to information or understanding that is shared among the members of a group. The idea of mutual knowledge is particularly relevant to understanding the value of groupwork because the greatest potential of groupwork stems from the assumption that different group members have different knowledge or ideas to contribute (Cohen, 1994). While mutual knowledge might be implied in many cases of casual conversation, it is not so obvious in groupwork or problem-solving settings (Schwartz, 1995). Turning individual knowledge and understanding into mutual knowledge within a group is necessary to leverage the potential of groupwork into meaningful learning opportunities.

One of the challenges of teaching and studying collaboration in school settings is that students and classrooms often have long-established hierarchies of social and academic status (Engle & Conant, 2002; Kurth et al., 2002; Langer-Osuna, 2016), and some students are granted more influence or authority in classroom discussions compared to their peers (Engle et al., 2014; Langer-Osuna, 2016). In these cases, the knowledge of higher status students tends to become the shared knowledge within a group, and common ground is not negotiated through interaction but rather through deference to one or two students. Because of this, it is unclear how individual knowledge becomes mutual knowledge and, similarly, how teachers might teach students to establish common ground in service of group problem solving.

This study documents how high school students established mutual knowledge in a context where each person had different individual information relevant to completing a group task. The data come from a summer program in which students learned about biology and engineering. One of the summer activities required students to work together on a game in which each person wore lenses that transmitted different wavelengths (colors) of light. Students needed to identify a sequence of colors displayed on a computer screen by combining the respective view of each person. The fact that each student had different color-filtering lenses and, therefore, each student had information that others in the group did not, created a natural experiment to document how students coordinated their work to establish common ground. Our analysis was guided by two research questions: How did groups establish mutual knowledge through their conversations during the color vision game? What types of knowledge or understanding became shared among the group members? The dual attention on how and what knowledge gets generated reflects the importance of social activity not only for the sake of learners' interpersonal experiences but also for the substance of what they learn.

2. Coordinated activity, mutual knowledge, and interdependence

One way to conceive of collaboration in groupwork is via equitable participation in work that leads to shared understanding (Esmonde, 2009). Equity, in this context, refers to a fair distribution of participation opportunities. This does not necessarily mean that every student talks the same amount – “fairness” is determined according to context. The framing of collaboration in terms of opportunities to participate in a group's work is grounded in a theory of learning as participation (Lave & Wenger, 1991; Rogoff, 1990). The learning of science through participation shifts emphasis from *knowing* science concepts to *doing* science. The National Research Council (2012) described eight practices inherent to learning science, which include asking questions, carrying out investigations, constructing explanations, and communicating information. Equitable and collaborative learning would require that all students in a group have opportunities to develop these practices.

The link between collaboration and learning to do science comes from how students participate in scientific practices through interaction. When instruction is designed to foster students' participation in practices such as gathering evidence or constructing an explanation, sometimes students can come to view the practice as an endpoint in itself rather than a means to do science (e.g., Berland & Hammer, 2012; Berland et al., 2016; Jimenez-Aleixandre et al., 2000; Schaffer & Resnick, 1999). Berland et al. (2016) described this distinction as the extent to which students' engagement in scientific practices “moves beyond the structural form of the practices” (p. 1103). Students might participate in scientific work to explain a phenomenon, support an argument, or help answer a question, or they might engage in the same work to meet some imposed criteria. All students can learn the practices of a discipline, but it is necessary to introduce students to these practices in ways that have utility beyond the immediate completion of a task (Edelson, 2001; Engle et al., 2014; Sengupta-Irving, T., 2014).

Two characteristics of groupwork are necessary for collaborative engagement in scientific practices. The first of these is interdependence, which refers to students' sense that their success depends on the other people within a group, and that their own contributions are necessary to others (Lotan, 2003). Beyond that, students must also develop the necessary skills to interpret and use their peers' contributions through coordinated activity. Coordinated activity is the means by which students can establish mutual knowledge (Gibbs Jr. & Mueller, 1990). Groups vary in the extent to which they coordinate their actions. Barron (2000), in a study comparing middle-grades students solving an open-ended mathematics problem, found that more successful groups coordinated their activity by responding to the correct ideas proposed by individuals, documenting those ideas, and reflecting on their utility. Other groups managed to generate correct ideas, but students in the less successful groups often ignored or rejected those proposals.

Instructional interventions that give students specific roles or scripts can help support coordinated activity. For example, group roles such as a materials manager or recorder can structure students' interactions so that everyone has a specific way to contribute to the group (Staples, 2007). Reciprocal teaching—where a pair of students alternates between an explainer and a listener—has been used with younger students with some success (Brown & Palincsar, 1989). Jigsaw activities are designed to let each person within a group develop expertise on a different aspect of an overarching task, so that students have to combine their individual knowledge to complete the task (Clarke, 1994). However, students do not always use group roles or scripts in ways that support coordinated activity and mutual knowledge. In fact, students sometimes invoke group roles to disregard a peer's contribution and prevent it from becoming common ground for the group (DeJarnette, 2022). Groupwork interventions that take place alongside students' work on a task can scaffold students' interactions, but they are not always effective. This study provides a novel context to study groups' coordinated actions because each person within a group held different information integral to completing the task. Because groups would be

required to coordinate their work, we sought to investigate both how students established mutual knowledge in interaction and, consequently, what knowledge became shared as a result.

3. Data and methods

3.1. The color vision activity

Exploring how humans, and other animals, perceive color is one way that students can learn about how multicellular organisms function while also developing students' systems thinking (e.g., the interactions of the components of a system) and developing scientific practices such as developing models, carrying out investigations, and interpreting data (NGSS Lead States, 2013). Questions related to color vision create opportunities for interdisciplinary teaching and learning because the anatomy of the human eye—specifically, how it “sees” color—can be modeled relatively simply with digital or electronic components (e.g., [Silveira et al., 2020](#)). In humans and many other animals, including great apes, cone cells in the eye are of three different types, each sensitive to a different wavelength of light. Red, green, and blue light stimulates or creates an electrical signal in a subset of the cones. In reality, cones are sensitive to somewhat broad and overlapping ranges of light wavelengths, and our perception of the color of any given wavelength depends on our brains' comparison of the relative amount of electrical signal in the here-in referred to ‘Red, Green, and Blue cones’. For instance, stimulating the Green cones alone results in the perception of the color green, but stimulating the Red and Green (but not Blue) cones together does not result in the simultaneous sensation of both red and green color; rather the brain deduces what single wavelength could stimulate both red and green cones, and generates the color perception of yellow. Stimulating the Red and Blue (but not Green) cones together doesn't generate the simultaneous perception of red and blue, but rather a single, pink-to-purple color, magenta. Stimulating the Green and Blue (but not Red) cones together produces the sensation of a blue-green, or cyan, color.

The color vision activity was designed for students to work together to explore how the human eye perceives color, and consisted of playing a computer game in which a pre-programmed sequence of colored shapes on a computer screen were to be identified to earn points. During the color vision activity, each student within a group wore a pair of goggles, and each pair of goggles filtered out all but red, green, or blue light, respectively (herein referred to, for instance, as blue goggles). In a group of three, each student could detect one of the three colors while wearing the goggles. So, for example, if looking at a blue screen while wearing the goggles, only a student with blue goggles would see bright blue, and very similar to the white background (composed of red, green and blue); the other students in the group would see black because the goggles filter out all wavelengths except for blue light. The activity's learning objectives described that students needed to explain how a range of color perceptions are created by a computer that emits only red, green, and blue wavelengths of light. Relatedly, students needed to explain how the human eye discriminates colors of light, that is, how computers combine three wavelengths to “fool” the visual system and create the perception of any arbitrary color.

The color vision activity was divided into two parts. In the first part, students completed a set of “training” rounds to determine how individuals wearing different goggles would perceive six different colors including red, blue, green, magenta, yellow and cyan. The program, which ran on a computer, displayed one of these six target colors in a rectangular region across the top of the screen ([Fig. 1](#)). Students could select a color from a text-based dropdown menu and then, wearing the goggles, each student could share how they perceived the selected known color and get to see the pattern of squares below associated with the known color. For example, if students selected to view yellow in the top rectangle, then a student wearing the red goggles would see the rectangle as bright red, and



Fig. 1. A Sample Training Round of the Color Vision Activity. A dropdown box at the top of the screen allowed students to select a color displayed in the long rectangle. The smaller rectangles display the selected color's components. Yellow (the case depicted in the figure) contains both red and green light but no blue light (indicated by the rightmost box being black).

a student wearing the green goggles would also see bright green. A student wearing the blue goggles would see very dark blue or black, because yellow does not include any blue light.

To complete the training program, students needed to work as a group to record what each of the four group members observed for each of the six colors (Fig. 2). Each group had a single handout to record their work. The idea was that they could create a “key” to relate the pattern of the brightness seen by each of them to the color of the test rectangle. The students then moved on to the second part of the activity, where they played a color identification game that was timed and scored. In this game mode, students were given in text the name of a color and had to select the correct colored rectangle(s) from a panel of nine options. They repeated this activity multiple times as new panels were presented to them. Playing the game required coordinated deduction of the pattern observed among students, because each person could only perceive one of the three colors.

3.2. Participants and data

Students in this study worked in groups of four due to social distancing measures in place during the summer program. Students worked on the color vision activity—including the training rounds and game—for 45 min, the duration of which we videorecorded. During that time, each group of students stood around one laptop computer that ran the color vision program. We videorecorded two groups during the activity, who had consented to participate in the study (Table 1). The research was approved through the university's Internal Review Board, and students ranged in age from 16 to 17 years old. The activity occurred at the start of the second week of the 3-week summer program. The students listed in Table 1 did not know each other prior to the start of the program, except for a pair of sisters who both attended the program. At the time of this data collection, students had known each other for a week and had spent time working on other biology and engineering challenges together during that week.

We recorded each group of students by positioning two video recorders behind each group to record their conversations as well as the computer display and students' movements around the laptop. We used the recordings to produce a transcript for each group of students. To analyze the data, we used the transcripts in coordination with the video so that we could simultaneously attend to students' talk, the visual representation on the screen, and body language that would indicate who a speaker was directing a comment to and whether or not the comment was heard.

3.3. Analysis

We developed a coding scheme to analyze the types of contributions students' made in conversation, especially with respect to the degree to which they attended to one another's contributions. For each group, we divided the transcript into segments according to the different rounds of training and game play. Part 1 of the activity included a set of “training” rounds (one corresponding to each color in Fig. 2, except black and white), and part 2 included 10–12 rounds of the game. In each game round, students were prompted (by a display of text on the screen) with one of the six colors, and they needed to determine which squares were that color. Group 1 completed four training rounds in addition to 12 rounds of the game, for a total of 16 segments of transcript. Group 2 completed four training rounds and 11 rounds of the game, for a total of 15 segments of transcript.

The first author developed an inductive coding scheme that was informed by Barron's (2000, 2003) findings that the quantity of students' talk was less indicative of their coordinated success on a task than the degree to which students responded to and took up one another's ideas. The initial set of codes was intended to specify whether students offered information, asked questions of others, or directly reacted to comments made by their peers. In collaboration with a research assistant, the first author refined the initial coding scheme to create the set of codes described in Table 2. The first three codes in Table 2 were used to describe the different ways that students could contribute information to the group's conversation. We characterized students' contributions according to whether they offered new information, repeated or elaborated on something they had previously said, or repeated or elaborated on something a peer had previously said. In addition to the moves students used to offer information, we delineated four different ways that they could directly react to a peer's contribution—by acknowledging that contribution, confirming it, rejecting it, or clarifying it. These reacting moves varied in the nature and degree of a speaker's response. An acknowledgment, for example, was a short, straightforward response to indicate the previous speaker had been heard. A confirmation indicated some agreement. A rejection indicated disagreement or challenge towards a peer's statement, and a clarification indicated some degree of confusion or misunderstanding.

The code “incorporate prior information” was a discussion move that we defined to describe instances when a student within a group verbally summarized what they knew about how each color filter perceived different colors. The way this move surfaced in this study was specific to the design of the color vision activity and related to the fact that the activity was divided into multiple “rounds” of the training phase and activity phase. The move “incorporate” surfaced relatively infrequently—as we will share in the following section—but it was notable for how it allowed students to verbalize prior knowledge from multiple people within a group and make

| | | Actual color of shape on screen | | | | | | | |
|-----------------------------|-------|--|-------|------|--------|---------|------|-------|-------|
| | | Red | Green | Blue | Yellow | Magenta | Cyan | White | Black |
| Student 'photoreceptors' | Red | + | - | - | + | + | - | + | - |
| | Green | - | + | - | + | - | + | + | - |
| | Blue | - | - | + | - | + | + | + | - |
| | | + = shape appears bright - = shape appears dark | | | | | | | |

Fig. 2. Color discrimination key.

Table 1
Participants, including each participant's grade-level following the summer program.

| Group | Students and grade levels |
|---------|---|
| Group 1 | Maria (12), Aditi (10), Tanea (12), Ronnie (11) |
| Group 2 | Samuel (12), Brody (11), Madeline (12), Jordyn (12) |

Note: Aditi and Tanea were sisters and therefore knew each other before the summer program.

Table 2
Students' discussion moves to coordinate action during the color vision activity.

| Type of contribution | Definition |
|-------------------------------|---|
| Offer new information | A student makes a new statement (i.e., not a repeat of something previously said). |
| Repeat or elaborate self | A student directly repeats, or expands upon, a statement that they have previously made. |
| Repeat or elaborate a peer | A student directly repeats, or expands upon, a statement made by another person in the group. |
| Acknowledge a statement | A student reacts to something said by another person in the group by acknowledging that it's been heard (e.g., "oh, okay"). |
| Confirm a statement | A student reacts to something said by another person in the group by indicating their agreement (e.g., "yep"). |
| Reject a statement | A student reacts to something said by another person in the group by challenging or contradicting it (e.g., "are you serious?"). |
| Clarify a statement | A student reacts to something said by another person in the group by seeking clarification (e.g., "what's red for you?"). |
| Incorporate prior information | A student reminds the group of knowledge that has been shared in an earlier round (e.g., "for magenta, green can see bright, blue can see light grey, and red sees black"). |
| Ask for information | A student asks a question (e.g., "Is cyan on 2?"). |

that part of the group's mutual knowledge.

A research assistant and the first author coded the transcripts independently, using the codes described in Table 2, to check the reliability of our analysis. Our reliability was 90 %, calculated as the number of turns coded the same divided by the total number of turns coded. We resolved remaining disagreements through consensus and by making comparisons to other turns that we had coded in agreement. We coded only the turns of speech that served the functions described in Table 2, so not every turn received a code (i.e., students' off-task talk, and talk about the rules of the game, speech that was interrupted before a complete thought was shared). In group 1, we coded 297 out of 387 turns of speech (77 %). Most of the turns that did not get coded in this group came from the training phase, when the students elected to trade their goggles back and forth. We did not code interactions related to facilitating those exchanges. In group 2 we coded 158 out of 255 turns of speech (70 %). Most of the uncoded turns for group 2 came from the training round, when a teaching assistant worked closely with the group for the first round and facilitated the discussion.

In our final phase of analysis, we coded each group's discussion to document the quality of mutual knowledge that surfaced within each round of the activity. We completed this coding collaboratively, which occurred in two steps. The first step was a binary decision of whether, within the discussion for a given round of the activity, there was evidence of mutual knowledge (i.e., knowledge that became shared among the group members). One indicator of mutual knowledge was that a student in the group reacted directly to something another person said through one of the moves summarized in Table 2. Another indicator of mutual knowledge was that a student offered new information that implicitly incorporated a prior statement by another person in the group.

In cases where we saw evidence of mutual knowledge, we used the Toulmin (1958) model of argumentation to document the quality of the knowledge. Toulmin's model breaks down an argument into components including (1) a claim to be made, (2) the data used to support the claim, and (3) the warrant (i.e., justification) for the claim. These components allowed us to clarify the extent to which students verbalized explanations for the phenomena they observed during the color vision game. For example, students could agree on claims (e.g., the displayed color is cyan) without necessarily talking about why they agreed. When students discussed some of the data for their claims (e.g., I see bright on square 5, so it must be red), we coded the interaction as such. For us to code a discussion as including a warrant, we looked for a verbalized explanation of the connection between the data and claim (e.g., yellow appears bright to both red and green, so the color must be yellow).

One implication of our coding scheme is that our identification of mutual knowledge depended on ideas becoming verbalized by at least two group members. This gave us evidence of when students' contributions were heard and incorporated into a group's discussion, but it does not necessarily imply that knowledge that became shared at the group level was used by every individual. Also, in our coding for the quality of mutual knowledge that surfaced, we reemphasize that our purpose was to document the nature of ideas that became public within each group. There were likely cases in which the data or, respectively, warrants for students' claims were implied but not said out loud. We focused on ideas that became verbalized because the purpose of this study was to document how knowledge becomes shared during groupwork.

4. Findings

4.1. Students' use of discussion moves

Table 3 summarizes each group's performance in the color vision game and the types of discussion moves used by students in each group. Group 1 correctly selected the target color square(s) in 10 out of 12 rounds of the game. Group 2 correctly selected the target

color in 10 out of 11 rounds. Group 1 spent more time discussing each round than group 2. In group 1, the mean discussion length per round was 20 turns of speech, and discussions ranged from 5 turns (on the second to last game round) to 50 turns (on the third training round). In group 2, the mean length of discussion per round was 10 turns of speech, and discussions ranged from five turns (the second training round) to 16 turns (the sixth game round). Overall, group 1 had more extensive discussions about the task and more disagreement among the members of the group, while group 2 spent less time discussing the task and experienced less contention.

The percentages in Table 3 indicate the relative frequencies of each type of move included in our analysis, which help characterize the similarities and differences across the two groups. In both groups, students spent more time offering new information than doing anything else. This happened in the form of students describing what they saw on the screen or stating their responses to the game prompts. The frequency of offering new information is a natural outcome of the game design, in which each student within a group had a different view of the screen because of the different color-filtering lenses. Most students in each group participated in the discussion in almost every round of the activity. In group 1 there were two rounds of the game in which Samuel did not make any verbal comments because he was adjusting his goggles. In group 2, there was one round of the game in which Ronnie did not make any verbal comments, and there were four rounds when Aditi did not speak. Aditi was one of the youngest participants in the summer program, and she wore the same colored goggles as her older sister, who was in her group. Those factors may have impacted Aditi's experience in the group and help explain why she was less vocal. Overall, students' participation rates in each group's conversation were high.

Beyond offering new information, group 1 and group 2 varied in how they used other types of discussion moves. The students in group 1 spent more time repeating or elaborating their own prior comments, followed by instances of reacting to one another. The students in group 2 spent more time reacting, followed by the frequency with which they asked questions of each other. Across both groups, students infrequently repeated or elaborated something that someone else in the group had said. Instances of incorporating information were the least frequent across both groups. This observation is notable because we defined the "incorporating" code to describe instances in which students verbalized how different students' perspectives were related to each other. This type of move seemed important for how students would establish shared knowledge within the group, but they rarely verbalized those connections.

Table 4 disaggregates students' reacting moves and summarizes the total number of times each move was used within each group. We hypothesized that reacting moves—which, by definition, were the moves with which students acknowledged, confirmed, rejected, or clarified their peers' contributions—would be important markers of how students established interdependence in their work. These moves were relatively infrequent overall, as is reflected in Table 4, and their uses differed between the two groups. Reacting moves were meaningful to students' conversations not because of their frequency but because of the impact they could have on the content of students' conversations. In the following section, we will describe the work of each group in more detail to illustrate how different types of reactions, moves to incorporate prior knowledge, and questions, functioned to support students' shared knowledge and interdependence.

Finally, Table 5 summarizes the quality of mutual knowledge that each group established. Our unit of analysis for coding for mutual knowledge was a single round of the activity. In other words, each time a new color appeared on the screen for each group, we documented the nature of the mutual knowledge that surfaced in the group's discussion of that color. In both groups, there were a small number of instances in which we saw no evidence of mutual knowledge. In these cases, each person in the group may have made a statement, but there was not verbal evidence that students' statements connected to one another. In group 1, there were two instances in which the only mutual knowledge we could identify was the general claim (e.g., the color is green). There were five rounds in which students shared knowledge related to the data for their claim, and four rounds in which they shared a justification. Group 2 included more instances where students shared knowledge about the data for their claims but no warrant, and only one round in which the warrant for a claim became mutual knowledge.

The numbers in Table 5, in coordination with Tables 2 and 3, suggest some correlation between the types of conversational moves students used and the nature of knowledge that became shared within the group. Specifically, students in group 1 used more rejecting and clarifying moves towards their peers, which may have provoked them to verbalize warrants for their claims beyond stating their data. Students in group 2 more often confirmed one another's statements without requiring explanation. Additionally, a notable difference between group 1 and group 2 was that group 1 included a student who consistently used incorporating moves to verbalize patterns that the group had established, which served as warrants for the claims that students made about colors in the later rounds of the game. In the following section, we provide examples of interactions between students to illustrate the phenomena we observed.

Table 3
Summary of group performance and discussion moves.

| Group observations | Group 1 | Group 2 |
|--|---------|---------|
| Game slides answered correctly | 10/12 | 10/11 |
| Turns of speech coded | 297 | 158 |
| Mean discussion length (# of turns) | 20 | 10 |
| Percent turns offering new information | 25 % | 35 % |
| Percent turns repeating or elaborating self | 18 % | 6 % |
| Percent turns repeating or elaborating other | 7 % | 8 % |
| Percent turns reacting | 13 % | 18 % |
| Percent turns incorporating | 4 % | 1 % |
| Percent turns asking | 6 % | 14 % |

Note. The denominators in row 1 are different because group 2 inadvertently skipped one round of the game.

Table 4
Aggregate counts of different types of reacting moves.

| Type of reacting move | Group 1 | Group 2 |
|---------------------------|---------|---------|
| Acknowledging a statement | 6 | 6 |
| Confirming a statement | 7 | 12 |
| Rejecting a statement | 13 | 5 |
| Clarifying a statement | 13 | 5 |

Table 5
Quality of mutual knowledge within each activity round for each group.

| Quality of mutual knowledge | Group 1 | Group 2 |
|-----------------------------|---------|---------|
| Claim only | 2/16 | 1/15 |
| Data for a claim | 5/16 | 11/15 |
| Warrant for a claim | 4/16 | 1/15 |
| No mutual knowledge | 3/16 | 2/15 |

Note. Denominators refer to the total number of rounds of the activity, including training rounds and game rounds.

4.2. Discussion moves and their connections to the quality of mutual knowledge

Because the move “offer new information” was the most frequent relative to the other discussion moves, students' use of this move provided the basic structure to most of their conversations. [Table 6](#) provides an example of a typical interaction that surfaced in both groups, using mostly of moves to offer new information. The conversation in [Table 6](#) comes from group 2 during the third round of the color vision game. In this round, magenta was displayed on square 8. Tanea and Aditi both wore blue-filter goggles. Ronnie wore green-filter goggles, and Maria wore red-filter goggles. After Tanea announced the color they were required to identify, Maria, Ronnie, and Tanea each offered new information to state which squares they perceived as magenta. After Maria repeated herself, Tanea asked which square the group should select and Ronnie suggested square 8. Although the group did not directly react to one another's suggestions until the end of the discussion, they implicitly used a process of elimination to determine which of the squares could be magenta. Ronnie's suggestion of square 8 was the only square that all students shared in common.

In [Table 6](#), each student made their individual knowledge—in the form of what they could see—public. Although students did not directly react to each other, we can infer that they heard and used that information based on the narrowing down process and their final conclusion. The knowledge that students shared in this case was the set of data upon which they based their claim that magenta was displayed on square 8. Students built on the data that each person shared to narrow down their selection. What did not surface verbally, however, was any warrant for their claim. The implied explanation comes from how the different lenses perceived each color and how the combination of three different perspectives could be used for the group to determine the location of magenta on the screen. It is possible that students reasoned through this explanation on an individual basis, but that knowledge never became verbalized as mutual knowledge within the group.

There were a variety of ways in which the general pattern of communication from [Table 6](#) became elaborated into more complex interactions. The example in [Table 7](#) comes again from group 2, during the second to last round of the game. Students needed to locate the color green, which was displayed only on square 3. The conversation began similarly to that in [Table 6](#), with each student offering information about what they saw. Because those four pieces of information didn't clearly lead to a conclusion, Tanea asked a specific question about who wore the green goggles and, then, what Ronnie perceived through those goggles.

After Tanea clarified with Ronnie, the group selected both 3 and 8 as representing green in the game (in fact, only square 8 was green). Tanea's questions about who wore green filters and, then, which squares looked white through the green filters, had the potential for students to talk about the implied warrant for their claim—how the different color filters worked together to help the students perceive colors on the screen. Doing so could have led the group towards more explanation, beyond stating what each person

Table 6
An example from group 2 attempting to locate magenta in the color vision game.

| Speaker | Turn of speech | Code |
|---------|--|---------------------|
| Tanea | Magenta | |
| Maria | So 4, 8, 6, and 9 are like whitish color for me. | Offer – New |
| Ronnie | For me it's either 5 or 8. | Offer – New |
| Tanea | Yeah, 3, 5, or 8 are like... | Offer – New |
| Maria | I think it is between 4, 8, 6, and 9. | Offer – Repeat self |
| Tanea | So which one? | Ask |
| Ronnie | Eight? I guess? | Offer – New |
| Tanea | Eight. | React – Acknowledge |

Note. Magenta was displayed on square 8. Tanea and Aditi wore blue goggles. Ronnie wore green goggles, and Maria wore red goggles.

Table 7
An example from group 2 attempting to locate green in the color vision game.

| Speaker | Turn of speech | Code |
|---------|---|------------------------|
| Tanea | Three is the only one that is green for me. | Offer – New |
| Aditi | Yeah, I see 3 as green. | Offer – Repeat other |
| Ronnie | For me, 8 is green. | Offer – New |
| Maria | I see 3 as blue. | Offer – New |
| Tanea | Who has the green filters? | Ask |
| Ronnie | Me. | Offer – New |
| Tanea | Okay, so what is white for you? | Ask |
| Ronnie | Three and 8. | Offer – Elaborate self |
| Tanea | Three and 8? | React – Clarify |

Note. Green was displayed on square 3. Tanea and Aditi wore blue goggles. Ronnie wore green goggles, and Maria wore red goggles.

saw. However, by framing the question only about the green filter, the group did not account for the fact that green-filtered lenses would perceive more colors than only green—including cyan and yellow. Ronnie's individual perspective became mutual knowledge within the group, but his observation alone was not enough to fully justify the group's claim. It would have been necessary to coordinate that knowledge with that of others in the group correctly to deduce that only square 3 was green.

Group 1 generally had longer discussions than group 2, and the knowledge shared among the group was more complex in some cases. The example in [Table 8](#) comes from group 1 during the seventh round of the game, when cyan appeared on square 2. Madeline wore red-filtering lenses. Samuel and Jordyn both wore blue-filtering lenses, and Brody wore green-filtering lenses. Jordyn spoke first, to offer her claim that cyan appeared on 2. In the following turn, Madeline elaborated on Jordyn's claim to extend the possible options for cyan to include square 6 in addition to square 2. We coded Madeline's move that as “elaborate” because she reacted directly to Jordyn comment and extended Jordyn's claim into her own. In the following two turns, Brody and Samuel – and then Madeline, again – each similarly elaborated on the prior responses. After they completed this process, Madeline and Brody incorporated information from earlier rounds to confirm the group's choice.

Brody and Madeline's use of incorporating moves in [Table 8](#) (i.e., “red sees black,” and “we see white”) re-surfaced the group's prior knowledge of how each of the color filters perceived the different colors. The information that Brody and Madeline offered, and the elaboration that followed, provided the warrant for their claims they did about which squares were cyan. There was a subtle but important difference here in the quality of knowledge that became common ground within the group. Compared to our prior examples, in which students shared data about what they saw without vocalizing its significance, the knowledge shared here was explanatory in nature. The content of students' discussions better reflected the intended goals of the lesson, which included engaging in the scientific practice of explaining a phenomenon.

We share a final example from group 1 to illustrate how complicated it was for students to integrate the information from the color filters ([Table 9](#)). This excerpt comes from the ninth round of the color vision game. Students were prompted to locate the magenta squares on the screen, which were squares 4, 7, and 8. The excerpt begins right after the monitor switched from the prior round. By the end of this game round, the group selected square 4, but Brody disagreed with his group about this for most of the conversation.

Brody initiated the group's conversation by incorporating prior information that the group had recorded during the training round of the activity. When he noted, “I see bright, blue sees light grey, red sees black,” Brody was reading from a handout on which he had recorded notes from the group. His statement was an attempt to provide a general basis on which the group could make a claim about where magenta was displayed. Following that, Jordyn and Samuel (both wearing blue-filter goggles) each offered new information noting what they observed to be the magenta squares on the screen. Brody did not directly respond to Jordyn or Samuel but instead asked Madeline a question. His question reflected another attempt to apply the group's prior information to the case at hand. Madeline's response led Brody to suggest that square 6 was magenta, which Madeline and Jordyn promptly clarified and, respectively,

Table 8
An example from group 1 attempting to locate cyan in the color vision game.

| Speaker | Turn of speech | Code |
|----------|---|-------------------------|
| Jordyn | Two? | Offer – New |
| Madeline | Cyan? It's either 2 or 6. | Offer – Elaborate other |
| Brody | Six for light blue. I see white, I see clear, so it's 2, 6, or 7. | Offer – Elaborate other |
| Samuel | It's 6 or 7. I think it's 6 or 7. | Offer – Elaborate other |
| Madeline | No it's not 7. It's 2 or 6. | Offer – Elaborate other |
| Jordyn | [Looking at Madeline] Two? I click 2? | Ask |
| Brody | Black sees... or – | |
| Madeline | Red sees black, and 2 and 6 are black, so. | Incorporate |
| Brody | [Pointing at Madeline] You see black. | Incorporate |
| Samuel | Is cyan light blue? | Ask |
| Brody | And we see white. So it's 2, 6, or 7. | Offer – Elaborate self |
| Samuel | It's 2. | Offer – New |

Note. Cyan was displayed on square 2. Jordyn and Samuel wore blue goggles. Brody wore green goggles, and Madeline wore red goggles.

Table 9
An example from group 1 attempting to locate magenta in the color vision game.

| Speaker | Turn of speech | Discussion move |
|----------|---|-------------------------|
| Brody | I see bright, blue sees light grey, red sees black. | Incorporate |
| Jordyn | Four, 7, and 8? | Offer – New |
| Samuel | All right so for blue it is 4, 7, and 8. | Offer – New |
| Brody | [To Madeline] What do you see as black? | Ask |
| Madeline | I see black as 1, 2, 5, and 6. | Offer – New |
| Jordyn | So is it 4, 7, 8? | Offer – Repeat self |
| Brody | It's six. | Offer – New |
| Jordyn | Six? | React – Clarify |
| Samuel | I see 4 – | |
| Madeline | No, it can't be 6. | React – Reject |
| Jordyn | No, it's not 6. | React – Reject |
| Madeline | No because magenta wasn't black was it? | Ask |
| Samuel | Six looks blue to me. | Offer – Elaborate self |
| Brody | [To Madeline] For you, magenta is black. | Incorporate |
| Madeline | Oh then okay. | React – Acknowledge |
| Brody | It's 2 and 6. | Offer – New |
| Jordyn | No. | React – Reject |
| Samuel | I got 4, 7, and 8. | Offering – Repeat self |
| Jordyn | I got 4, 7, and 8. | Offer – Repeat self |
| Brody | Blue sees grey for 4, 7 and 8. | Offer – Elaborate other |

Note. Magenta was displayed on squares 4, 7, and 8. Jordyn and Samuel wore blue goggles. Brody wore green goggles, and Madeline wore red goggles.

rejected.

Madeline's question, "magenta wasn't black, was it?" and Brody's response indicated the central tension within the conversation. Brody's incorporating move was an attempt to make a reasoned inference about the color on the screen based on information that the group had synthesized. But there were two central problems with Brody's attempts. The first was that his records included errors, as reflected by Madeline's uncertainty about his claims of how red-filtered lenses should perceive magenta. Perhaps more importantly, other members of the group were uninterested in Brody's reasoning. Samuel and Jordyn made conclusions about the target color based on their individual perspectives. While discrepancies in the group's ideas could have been a site to revisit and clarify the general principles of color discrimination, that potential was not always realized.

5. Discussion

The discussion moves students used had implications for both how knowledge became shared within a group and what knowledge was shared. In the most straightforward cases, students offered information about what they saw and identified a target color via a process of elimination. This style of interaction—where students make statements rather than ask one another questions—is typical of students working in groups (DeJarnette & González, 2015) and is a contrast to whole-class discussions, in which teachers often lead students towards important ideas through a sequence of guiding questions. Notable in the present study is the fact that students did not have to engage each other directly through posing questions or re-voicing—discourse strategies that are often used in classrooms—to hear and incorporate one another's ideas. Instead, students had a range of ways to build on one another's contributions. Sometimes this included direct responses such as acknowledgments or clarifications, but other times students offered new ideas that implicitly built on what someone had previously said. Given the emphasis placed in prior research on students responding directly to one another (e.g., Barron, 2000), it is notable that students can establish common ground without making this responsiveness explicit through their talk.

The key differences between the two groups in our study were related to (a) how they responded to one another's statements and (b) the substance of their talk in relation to the arguments they co-constructed. In group 1—where students consistently clarified or rejected one another's claims—they provided more warrants for their claims. In other words, students in group 1 provided rationales based on the underlying scientific principles. Students engaged in the scientific practice of explaining in a way that moved beyond its structural form (Berland et al., 2016), because they used these explanations towards a solution to the problem they were presented with. It is also true, however, that explanatory knowledge usually became verbalized because someone instigated it—usually Brody. Without that provocation, it is unclear whether the rest of the group would have made explanations of the principles of color vision part of their conversation.

In group 2, where students most often affirmed one another's statements without clarifications or challenges, claims were often supported by data (e.g., "I see dark") but rarely with warrants (e.g., "magenta appears dark through green filters"). In most cases, group 2 still met the objective of the game—the correct identification of colors. The work in this group reflects a tension of teaching science, which is how to balance the learning of content knowledge with engagement in scientific practices (Lehrer & Schauble, 2006; National Research Council, 2012). While a learning goal of the color vision activity was for students to explain the phenomenon of color discrimination, the product of the activity was to identify the colors on the screen. It is almost certain that at least some of the students in the group used the principles of color vision to synthesize the data shared by each person and to translate it into a response. But, it is difficult to know who participated in—and who had access to—that work, because it was rarely verbalized within the group.

There are limitations to studying students' coordinated activity and mutual knowledge through discussions, because much of the work that happens in classrooms is not verbalized (e.g., O'Halloran, 2000). Our definition of mutual knowledge assumes it is that knowledge that becomes verbalized in discussion. Additionally, our focus on students' groupwork and conversation prioritizes a view of learning through group participation and, consequently, obscures an alternative perspective of learning as an act of individual cognition. While we can describe the nature of knowledge that became apparent in student's talk, we cannot say the extent to which students might have reflected on explanations for color vision on an individual basis.

Moreover, in many contexts there is not necessarily intrinsic value to verbalizing mutual knowledge. In everyday conversation, it is more typical that what counts at mutual knowledge remains implicit (Schwartz, 1995). The central question relates to cultivating equitable groupwork environments and whether all students have fair opportunities to participate (Esmonde, 2009) when part of the work remains implicit. Knowledge and ways of reasoning can be "taken as shared" when they become normalized within a classroom community and, consequently, are implied rather than made a point of discussion (Cobb et al., 2001; Yackel & Cobb, 1996). However, for ideas or practices to be taken as shared does not mean that they are in fact shared by all students within a classroom. Our attention to ideas that students verbalized helps give a consistent view of knowledge that all students within a group had access to.

To achieve the coordinated action that can lead to mutual knowledge, members of a group need to believe that a task requires the contributions of multiple people. The case of the color vision activity created a useful experiment in which the contributions of multiple people were objectively necessary, because each person within a group had different information necessary for solving the task. This set-up is similar to existing groupwork interventions, such as jigsaw groups (Clarke, 1994). However, this model has limitations in its transferability to the real world. In real life, students' prior knowledge and experience is not parsed out so cleanly, and students often fall into hierarchies of status and achievement that foster imbalance (Esmonde, 2009). Therefore, it is necessary to consider what aspects of an intervention like the color vision activity might be adopted for teaching students to work productively in groups more broadly.

Students may benefit from more explicit instruction and scaffolding on the types of discussion moves that can be useful during groupwork. Research on teaching and teacher education has developed a vast repertoire of conversational moves that teachers and use to promote productive discussions (Chapin et al., 2009; Michaels & O'Connor, 2015). Specific talk moves among students are less studied and rarely taught in an explicit way. The use of group roles or scripts can be a way to introduce students to a variety of talk moves or, more generally, different ways to position oneself within a group. Although the use of group roles can sometimes feel unnatural for students, practicing such roles for a period of time can provide conversational tools that students later adopt in more fluid ways.

6. Conclusion

This study documented the work of two groups of high school students working in groups on a color vision activity, where students within a group each wore different color-filtering lenses and needed to integrate their individual perspectives to draw conclusions about different color displays. We sought to investigate how students established mutual knowledge through their talk and, relatedly, what types of knowledge became shared through this process. Students primarily shared observational information from their individual perspectives, which was efficient for completing the task but less effective towards the goal of explaining color discrimination. Students were more inclined to construct explanations of color discrimination when they used discussion moves like asking each other questions, reacting to each other's statements, and incorporating multiple perspectives. Analysis of mutual knowledge within a group requires attention to both how this knowledge gets constructed and what type of knowledge gets constructed. Instructional interventions on this aspect of groupwork should attend to both of these features.

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